# Sheet 3

**P7**

a)

$$\left.\begin{aligned} R_{HR} &= 1 \\ \Lambda &= 0.02 \\ R_{OC} &= 0.8 \end{aligned}\right\} \Rightarrow \quad G = \frac{1}{\sqrt{R_{HR}(1-\Lambda)R_{OC}}} = 1.129$$

b) $\langle N \rangle = 0.9 \cdot 1.38 \cdot 10^{26}\, m^{-3} = 1.242 \cdot 10^{26}\, m^{-3}$

$$\langle N_2 \rangle = \frac{\sigma_a}{\sigma_a + \sigma_e} \langle N \rangle + \frac{1}{L} \frac{\ln G}{\sigma_a + \sigma_e}$$

$$= \frac{0.156}{0.156 + 2.31} \cdot 1.242 \cdot 10^{26}\, m^{-3} + \frac{1}{0.04\, m} \cdot \frac{\ln 1.129}{(0.156 + 2.31) \cdot 10^{-20}\, cm^2}$$

$$= \underline{\underline{9.087 \cdot 10^{24}\, m^{-3}}}$$

$$\langle N_1 \rangle = \langle N \rangle - \langle N_2 \rangle = \underline{\underline{1.151 \cdot 10^{26}\, m^{-3}}}$$

$$\eta_{abs} = 1 - e^{-\sigma_a(\lambda_p) \cdot \langle N_1 \rangle \cdot L}$$

$$= \underline{\underline{0.970}}$$

P7

c)

$$I_p^{sat} = \frac{hc}{\lambda_p(\sigma_a(\lambda_s)+\sigma_e(\lambda_s))\tau} = 9{,}0\ \frac{kW}{cm^2}$$

$$I_{th} = \frac{I_p^{sat}}{\eta_{abs}}\left(\ln G + \sigma_a \langle N\rangle L\right)$$

$$= \frac{9\frac{kW}{cm^2}}{0.97}\cdot\left(\ln 1.129 + 0.156\cdot 10^{-20}\,cm^2\cdot 1.242\cdot 10^{26}\,m^{-3}\cdot 40\,mm\right)$$

$$= 8.32\ \frac{kW}{cm^2}$$

$$\Rightarrow P_{th} = I_{th}\cdot A$$

$$= I_{th}\cdot \pi\cdot(1\,mm)^2$$

$$= \underline{\underline{261\,W}}$$

$$\eta_s = \frac{\lambda_p}{\lambda_s}\cdot\frac{T_{oc}}{2\ln G}\cdot\eta_{abs}$$

$$= \frac{941}{1029}\cdot\frac{0.2}{2\ln 1.129}\cdot 0.97$$

$$= \underline{\underline{0.731}}$$

P8)

a)

$$\ln G = \ln \frac{1}{\sqrt{R_{HR}(1-\Lambda)R_{oc}}}$$

$$= -\frac{1}{2}\ln\left[R_{HR}(1-\Lambda)R_{oc}\right]$$

$$= -\frac{1}{2}\ln\left[R_{HR}R_{oc}\right] - \frac{1}{2}\ln(1-\Lambda)$$

$$= +\ln G_0 - \frac{1}{2}\ln(1-\Lambda)$$

$$\eta_{s,abs} = \frac{\eta_s}{\eta_{abs}} = \frac{\lambda_p}{\lambda_s}\frac{-\ln(1-T_{oc})}{2\ln G}$$

$$\Rightarrow \boxed{\frac{1}{\eta_{s,abs}} = \frac{\lambda_s}{\lambda_p}\cdot\frac{2\ln G}{-\ln(1-T_{oc})} = \frac{\lambda_s}{\lambda_p}\cdot\frac{2\ln G_0}{-\ln(1-T_{oc})} - \frac{\lambda_s}{\lambda_p}\frac{\ln(1-\Lambda)}{T_{oc}}}$$

$$\boxed{\overset{T_{oc}\ll 1}{\approx} \frac{\lambda_s}{\lambda_p}\cdot\frac{2\ln G_0}{T_{oc}} - \frac{\lambda_s}{\lambda_p}\frac{\ln(1-\Lambda)}{T_{oc}}.}$$

P9

b) For $\Lambda \ll 1$ and $T_{oc} \ll 1$ and $R_{HR} = 1$ we find

- $\ln G_0 = \ln \frac{1}{\sqrt{1-T_{oc}}} = -\frac{1}{2} \ln(1-T_{oc}) \approx \frac{1}{2} T_{oc}$

- $\ln(1-\Lambda) \approx -\Lambda$

$$\Rightarrow \frac{1}{\eta_{s,abs}} \approx \frac{\lambda_s}{\lambda_p} \cdot \frac{\cancel{2} \cdot \frac{1}{\cancel{2}} \cancel{T_{oc}}}{\cancel{T_{oc}}} - \frac{\lambda_s}{\lambda_p} \cdot \frac{-\Lambda}{T_{oc}}$$

$$= \frac{\lambda_s}{\lambda_p} + \frac{\lambda_s}{\lambda_p} \cdot \frac{\Lambda}{T_{oc}}$$

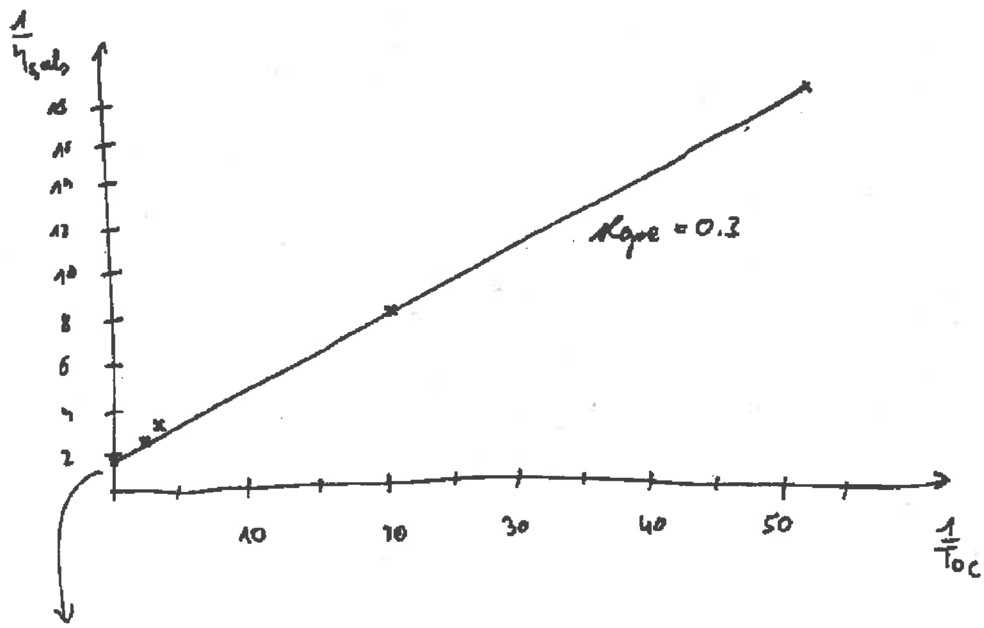
$$= \frac{1}{\eta_0} + \frac{\Lambda}{\eta_0} \cdot \frac{1}{T_{oc}}$$

$\Rightarrow \eta_0 = \frac{\lambda_s}{\lambda_p}$ in this case.

P8

c)

| $R_{OC}$ | $T_{OC}$ | $\frac{1}{T_{OC}}$ | $\eta_{s,abs}$ | $\frac{1}{\eta_{abs}}$ |
|---|---|---|---|---|
| 60% | 0,4 | 2,5 | 0.4 | 2.5 |
| 70% | 0,3 | 3,33 | 0.334 | 2.99 |
| 95% | 0.05 | 20 | 0.129 | 7.75 |
| 98% | 0.02 | 50 | 0.06 | 16.7 |




⇒ 1.) $\frac{1}{\eta_{s,abs,0}} = 1.87 \rightarrow \underline{\eta_{s,abs,0} = 53,5\%}$

2.) $0,3 = \frac{\Lambda}{\eta_0} \Rightarrow \Lambda = \eta_0 \times 0.3 = 0,535 \times 0.3 = 0,16$

A9

a)

$$\left.\frac{\partial\langle\phi\rangle}{\partial t}\right|_{spont} = \int \frac{\partial\tilde{\phi}}{\partial t}d\lambda$$

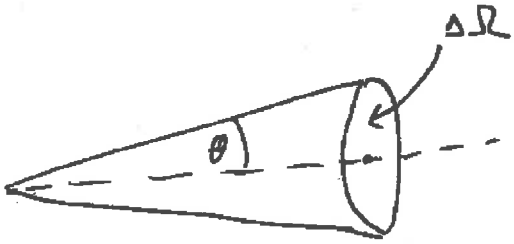
$$= \int c\,\sigma_e(\lambda)\langle N_2\rangle \frac{\Delta\Omega_s}{4\pi}\cdot\frac{8\pi n^2}{\lambda^4}d\lambda$$

$$= \frac{\Delta\Omega_s}{4\pi}\cdot\langle N_2\rangle\cdot\underbrace{8\pi c n^2\cdot\int\frac{\sigma_e(\lambda)}{\lambda^4}d\lambda}_{\frac{1}{\tau_{21}}}$$

$$= \frac{\Delta\Omega_s}{4\pi}\frac{\langle N_2\rangle}{\tau_{21}} \qquad \text{qed.}$$

P9

b) • $\theta = \dfrac{\lambda}{\pi w_0 n}$




• $\Delta\Omega = 2\pi\,(1-\cos\theta)$

$\approx 2\pi\left(1-\left[1-\frac{1}{2}\theta^2\right]\right)$

$= \pi\theta^2$

$\underline{\Delta\Omega = \dfrac{\lambda^2}{\pi w_0^2 n^2}}$

• $A_{mode} = \pi w_0^2$

• $\tilde{I}_{s,0} = \dfrac{\Delta\Omega_s}{4\pi}\cdot\dfrac{8\pi n^2 hc^2}{\lambda^5}$

$\Rightarrow \tilde{P}_{mode} = \tilde{I}_{s,0}\cdot A_{mode} = \dfrac{\cancel{\lambda^2}}{4\cancel{\pi^2}\,\cancel{w_0^2}\,\cancel{n^2}}\cdot\dfrac{8\cancel{\pi}\cancel{n^2}hc^2}{\cancel{\lambda^5}\,\lambda^3}\cdot\cancel{\pi w_0^2} = \dfrac{2hc^2}{\lambda^3}$

$\Rightarrow P_{mode} = \dfrac{2hc^2}{\lambda^3}\,d\lambda = \dfrac{2hc^2}{\lambda^2}\,\dfrac{d\lambda}{\lambda} = 2h\nu^2\,\dfrac{d\nu}{\nu} = \underline{\underline{2h\nu\,d\nu}}$ q.e.d.

$\Rightarrow$ each frequency component contribute one photon $h\nu$ to each polarization of the mode.